PI-D and I-PD Controller Design for a Two Loop Lateral Missile Autopilot in Pitch Plane

Biraj Guha

Abstract—The present work is an approach towards designing of the PI-D and I-PD controller for a two loop lateral missile autopilot system of the surface to surface tail controlled missile in pitch plane, characterized by the dynamics involving non-minimum phase zero. An autopilot is an automatic control mechanism for keeping the spacecraft in desired flight path. Conventional tuning method has been implemented for determining the controller tuning constants. Time response and frequency response of the designed autopilot system in pitch plane for a flight condition have been evaluated and the performances are presented. The frequency domain analysis of the designed control system has been carried out by opening the critical points on autopilot loops; the critical gain margin and critical phase margin of the system have been formulated. The effect of aerodynamic parameters variation on the performance of the designed autopilot system has been analyzed by Kharitonov's theorem. A study on the effect of external disturbance on the designed system has also been carried out in the present work.

Index Terms— Autopilot, Non-minimum phase, Missile, Pitch plane, PI-D controller, I-PD controller, Gain margin

1. INTRODUCTION

A systematic methodology for linear design in frequency domain of lateral autopilot for a class of guided missiles has been proposed in [1] and the design situation is considered where the missile actuator parameters (natural frequency ω_a and damping ratio ξ_a) are given and the airframe environment parameter are represented by the aerodynamic parameters T_a, m_η, σ and ω_b [Table 1]. When the autopilot controls motion in the pitch and the yaw plane, they are called lateral autopilot. The lateral autopilot in a guided missile is a servo system delivering lateral acceleration according to the demand from the guidance computer [7]. The autopilot configuration as developed in [1] has been utilized in the work [2] where the design objective is to develop the two loop missile autopilot configuration utilizing PI and PID control action employing the conventional Ziegler-Nichols method of controller design as a finite steady state error existed in the method prescribed in [1]. The PI controlled autopilot and the PID controlled autopilot as developed in [2] tracks the unit step response accurately with zero steady state error [Fig.4], however, the maximum percentage overshoots are large enough for both the situations. The present work utilizes two loop autopilot configuration as developed in [2] and employs PI-D and I-PD control action. The time and frequency response of the designed lateral autopilot as obtained in present work have been compared with the PID controlled autopilot system which has been designed in [2].

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TABLE 1
System Design Parameter Identification

Symbol	Parameter Identification	System Design Parameters
T_a	Incidence lags of airframe, sec	0.36
ω_b	Weathercock frequency, rad/sec	11.77
M_η	Moment derivative due to elevator deflection, sec^{-2} (Semi non-dimensional form)	-53.0
ξ_a	Damping ratio of actuator	0.6
K_b	Airframe aerodynamic gain, sec^{-1}	-9.91
ω_a	Natural frequency of oscillation of Actuator, rad/sec	180
σ	A quantity whose inverse determines the location of non-minimum phase zero in s-plane	σ^2=0.00029

2. PID CONTROLLER DESIGN

The control signal generated by a PID controller consists of proportional error signal added with derivative and integral of the error signal. The proportional term of controller is concerned with the current state of the process variable. The integral term when added with proportional term, accelerates the movement of process toward the set point and often eliminates the residual steady state error [8]. The rate of change of process error which is the differential slope of error over time is multiplied with derivative gain and the derivative control action may be utilized to reduce overshoot of response.

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2.1 PID Control Action

The PID controlled flight path rate demand autopilot in pitch plane with unity feedback derived from lateral autopilot with one accelerometer and one rate gyro in pitch plane as developed in [2] has been presented (Fig.1). Undershoot at the beginning of time response due to non-minimum phase zero has been improved in three loop autopilot configuration presented in [5]. Ziegler-Nichols tuning method has been utilized for determining the tuning constants of the controller.

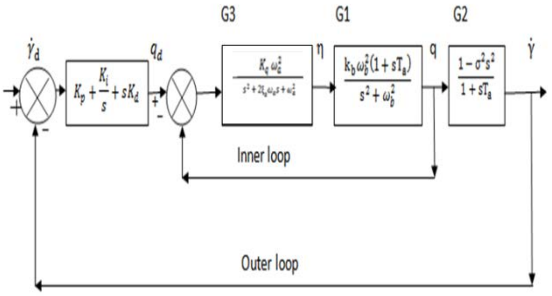

Fig.1. PID controlled two loop lateral autopilot configuration

Characteristic equation of two loop PID controlled autopilot system is given in (Eqn.1)

$$T_a s^6 + (1 + 2T_a \xi_a \omega_a) s^5 + (2\xi_a \omega_a + T_a \omega_a^2 + T_a \omega_b^2 - \sigma^2 k_p T_a - \sigma^2 k_d M) s^4 + (\omega_a^2 + \omega_b^2 + T_a k_d M + 2T_a \xi_a \omega_a \omega_b^2 + M T_a^2 - \sigma^2 k_p M - \sigma^2 k_i T_a) s^3 + (2\xi_a \omega_a \omega_b^2 + 2M T_a + T_a k_p M - \sigma^2 k_i M + k_d M) s^2 + (M + k_p M + \omega_a^2 \omega_b^2 + T_a k_i M) s + M k_i = 0$$

Where $M = k_b k_q \omega_a^2 \omega_b^2$ (1)

The tuning constants of PID controller as obtained by Ziegler-Nichols closed loop tuning, are given [Table 2]. Step response curve has been presented [Fig.4] and time domain specifications have been formulated [Table 3]. As seen, overshoot is large enough that needs to be limited.

TABLE 2
PID Tuning Constants

Proportional gain, K_p	Integral gain, K_i	Derivative gain, K_d
9.93	206.87	0.12

2.2 PI-D Control Action

Modification has been made on the PID control action in order to avoid the set point kick phenomenon with the same values of tuning constants obtained by conventional Ziegler-Nichols tuning method. The derivative control action may be operated only in the feedback path so that differentiation occurs only on the feedback signal and the corresponding control scheme is known as the PI-D control which has been illustrated in (Fig.2).

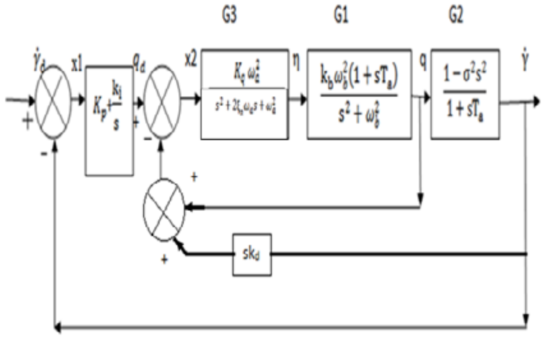

Fig.2. PI-D controlled two loop lateral autopilot configuration

Characteristic equation of two loop PI-D controlled autopilot system is given in (Eqn.2)

$$s^5 + \{\omega_a (2\xi_a - k_d k_b k_q \omega_a \omega_b^2 \sigma^2)\} s^4 + (\omega_a^2 + \omega_b^2 - k_p k_b k_q \omega_a^2 \omega_b^2 \sigma^2) s^3 + \{\omega_a \omega_b^2 (2\xi_a + k_b k_q \omega_a k_d - k_b k_q \omega_a k_i \sigma^2)\} s^2 + \{\omega_a^2 \omega_b^2 (1 + k_p k_b k_q)\} s + k_i k_b k_q \omega_a^2 \omega_b^2 = 0 \quad (2)$$

Step response curve has been presented [Fig.4]. In the present situation overshoot is larger with compare to the previous [Table 3].

2.3 I-PD Control Action

PID control and PI-D control involve a step function in the manipulated signal in accordance with the reference input which is a step function [3]. Such a step change in the manipulated signal may not be desirable in many occasions. Therefore, it may be beneficial to move the proportional action and derivative action to the feedback path so that these actions affect the feedback signal only (Fig.3).

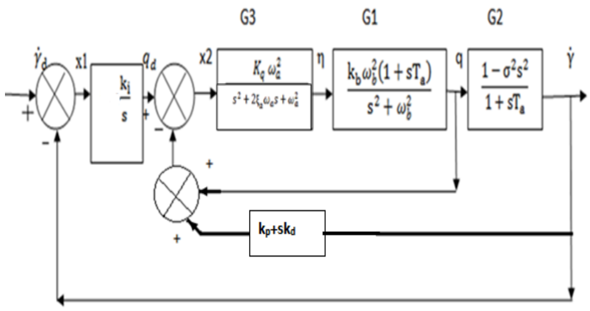

Fig.3. I-PD controlled two loop lateral autopilot configuration

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Characteristic equation of two loop PI-D controlled autopilot system is given in (Eqn.3).

$$s^5+\{\omega_a(2\xi_a-k_dk_bk_q\omega_a\omega_b^2\sigma^2)\}s^4+(\omega_a^2+\omega_b^2-k_pk_bk_q\omega_a^2\omega_b^2\sigma^2)s^3+$$
$$[\omega_a\omega_b^2\{2\xi_a+T_ak_bk_q\omega_a+k_bk_q\omega_a(k_d-k_i\sigma^2)\}]s^2+$$
$$\{\omega_a^2\omega_b^2(1+k_bk_q+k_pk_bk_q)\}s+k_ik_bk_q\omega_a^2\omega_b^2=0 \quad (3)$$

It is observed that I-PD controlled missile autopilot achieves improved transient response transient as it is limited to 17.5 percent [Table 2]. Step response curve has been presented [Fig.4].

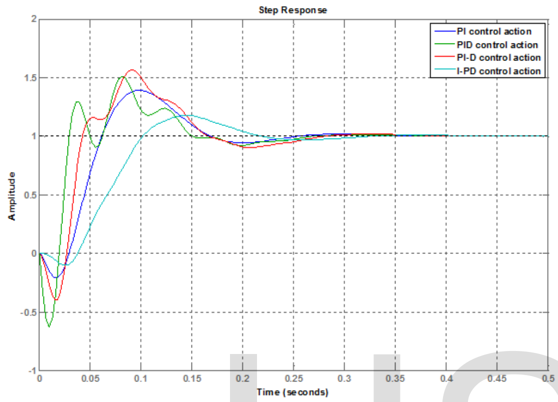

Fig.4. Step response of the designed autopilot system

Time response of the PID, PI-D and I-PD controlled autopilot in accordance with unit step has been formulated [Table 3].

TABLE 3
Time Response of Designed Autopilot System

Controller type	Percentage maximum overshoot	Rise time (Sec)	Peak time (Sec)	Settling time (Sec)	Steady state gain
PID	50.9	0.008	0.082	0.24	1
PI-D	56.6	0.011	0.091	0.26	1
I-PD	17.5	0.050	0.144	0.29	1

2.4 Frequency Domain Analysis

The frequency response of the flight path rate demand autopilot in pitch plain has been carried out by opening two critical points namely X1 and X2 separately. As a result, there will be a pair of gain margins, phase margins, gain crossover frequencies and phase crossover frequencies. The minimum of these have been taken into consideration as critical gain margin and critical phase margin.

It is seen that improved frequency response has been achieved in I-PD controlled autopilot system when the loop is opened at point X1, however, opening the loop at critical point X2 constitute the same result in all the three situations which has been presented [Table 3]. The Critical values have been obtained when the loop is broken at point X2 for the three situations and the fastest loop is the outer loop.

TABLE 4
Frequency Response of Designed Autopilot System

Controller type	Loop breaking point	Gain margin (dB)	Phase margin (deg)	Gain crossover frequency (rad/Sec)	Phase crossover frequency (rad/Sec)
PID	X1	2.33	35.5	34.1	135
	X2	1.77	29.3	63.5	138
PI-D	X1	5.14	29.3	29.9	99.7
	X2	1.77	29.3	63.5	138
I-PD	X1	7.68	51	17.2	42
	X2	1.77	29.3	63.5	138

The frequency response of I-PD controlled lateral autopilot is illustrated in Fig 5-A and Fig 5-B.

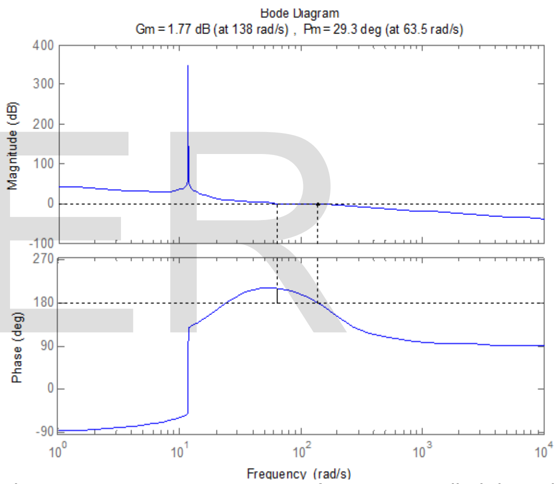

Fig.5-A. Frequency response of I-PD controlled lateral autopilot opened at point x1

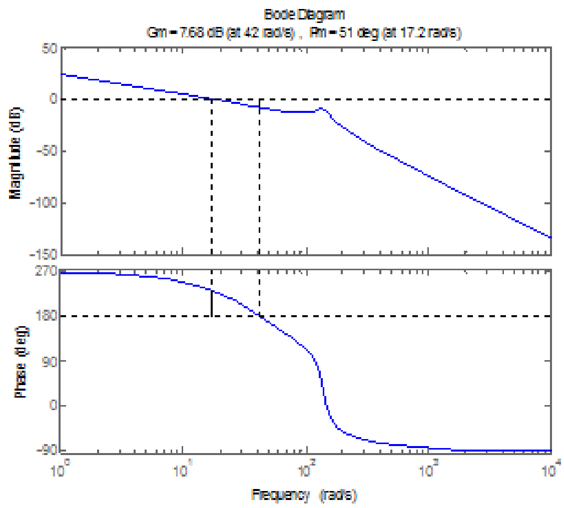

Fig.5-B. Frequency response of I-PD controlled lateral autopilot opened at point x2

3. A PRELIMINARY STUDY ON THE UNCERTAINTY OF SYSTEM PARAMETERS OF I-PD CONTROLLED AUTOPILOT BY KHARITONOV'S METHOD

Kharitonov's theorem is a result used in control theory to assess the stability of a dynamical system when the physical parameters of the system are not known precisely. The variations in the aerodynamic parameters T_a, K_b, ω_b, σ^2 of the I-PD controlled autopilot as developed in the present work are considered.

An interval polynomial is the family of all polynomials:

$$P(S) = P_n s^n + P_{n-1} s^{n-1} + P_{n-2} s^{n-2} + \ldots + P_1 s^1 + P_0 \quad (4)$$

Where each coefficient $P_i \in R$ can take any value in the specified intervals, $M_i \leq P_i \leq N_i$ and 'M_i' and 'N_i' are lower and upper specified range respectively of the corresponding coefficient. It is also assumed that the leading coefficient cannot be zero, i.e. $0 \notin [M_n, N_n]$.

Kharitonov's theorem has been applied on the characteristic equation (eqn-3) of I-PD controlled lateral missile autopilot system. Variations in the range of ± 10% on four aerodynamic parameters (K_b, σ^2, ω_b, T_a) have been evaluated and stable performance has been achieved (Fig.6).

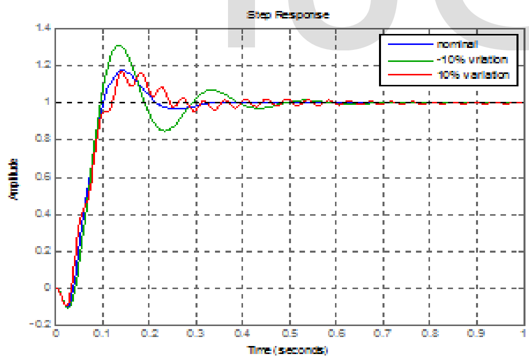

Fig.6. Step responses of the I-PD controlled autopilot system for ±10% parametric variation

Unit step responses of the system for ± 10% parametric variations have been formulated.

TABLE 5
Time Response for Parameter Variation

I-PD Controller	Percentage maximum overshoot	Settling time (Sec)	Steady state gain
Nominal	17.5	0.29	1
-10%	31	0.46	1
+10%	16.7	0.45	1

4. DISTURBANCE REJECTION ABILITY

Disturbance in a system may be due to unmodelled dynamics of the system or due to parametric variation or combination of the both. In the aerodynamic Control phase of the missile, external disturbances may cause deviations from the desired trajectory affecting the flight path rate, the control surface performance etc. In this section an attempt has been made to study and evaluate the effect of constant disturbance inputs at a point on the missile configuration (Fig 7). The dynamic response of the flight path rate $\dot{\gamma}$ due to unit step disturbance inserted in the I-PD controlled autopilot system for a flight condition have been studied and The response curves have been presented [Fig 8].

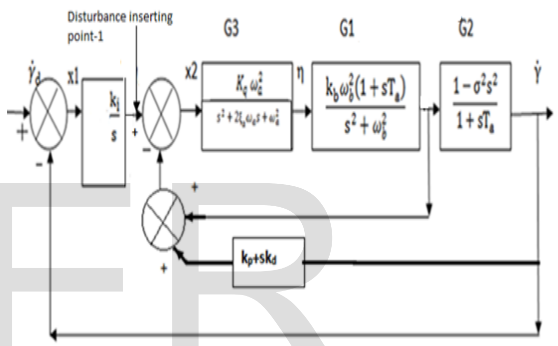

Fig.7. Disturbance inserting point on I-PD controlled autopilot

It is possible to attenuate the effect of external disturbance present in the aerodynamic control phase of the missile by employing PI controller in the two loop autopilot configuration of tail controlled missile in pitch plane and the designed autopilot system is capable of reducing flight pate rate to zero in steady state in response to external step disturbance.

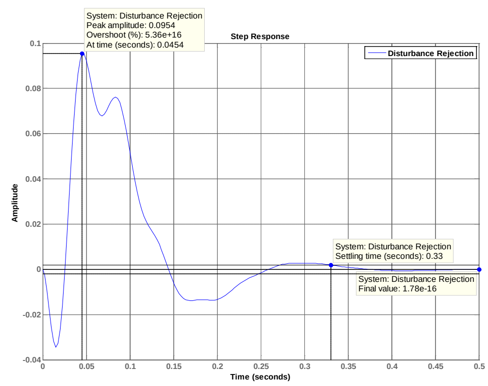

Fig.8. Disturbance rejection ability of the designed system

Although, identical response curves are obtained for the PID, PI-D and I-PD controlled autopilot system corresponding to step disturbance as in all the situations feedback loop consists of PID transfer function.

5. CONCLUSION

The performance characteristic of the two loop autopilot designed in [1] has been studied. Although, the transient performance specifications are satisfied but the steady state performances indicate that there exist steady state error in tracking unit step input. Designing PID, PI-D and I-PD controllers in the present work have made it possible to eliminate the steady state errors for step input. The results obtained for a flight condition of the designed missile autopilot system are presented. Frequency response studies have also been carried out to evaluate the stability performances of the PID, the PI-D and the I-PD controllers. The critical phase margins and the critical gain margins are evaluated by Ziegler Nichols design approach and the results are presented.

A study on the effect of parameter variations of the I-PD controlled autopilot has been carried out by applying the Kharitonov's method. The results obtained indicated stability robustness. An attempt has been made to investigate into the effect of external disturbances on the designed autopilot systems. The dynamic performance and disturbance rejection capability of the designed controlled autopilot system have been presented. It is observed that the designed controllers are capable to reject the effect of constant disturbance at the point completely.

Design of the PID controller for autopilot system can be evaluated by optimization techniques on the basis of performance index which may be envisaged as the future scope of work.

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